

A MODIFIED QUASISTATIONARY METHOD OF DESCRIBING THE KINETICS OF DRYING OF HYGROSCOPIC MATERIALS

G. I. Efremov

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The article considers the description of the kinetics of drying of hygroscopic materials on the example of wool fiber and grains of corn by a modified quasistationary method obtained in analyzing a diffusion flow in the material. The final relations are rather simple and agree with results of numerical solution of the system of hydrodynamic and heat and mass transfer equations. Satisfactory agreement is obtained between the predicted loss of moisture, rate of the process, and change in temperature with the time of the process and the experimental values.

Introduction. Processes of drying and wetting have found wide application in various areas of technology. Despite their great qualitative diversity, their kinetics can be described identically in the majority of cases, as shown by an analysis of experimental data. In contrast to stationary technological processes, here the value of the mass-transfer coefficient β changes with time τ . In the initial period there is a maximum concentration gradient, and diffusion proceeds very intensely. As the process proceeds, the concentration gradient decreases, diffusion slows down, and, when the gradient becomes equal to zero, the process terminates.

In accordance with the model of mass conduction [1], for diffusion in an isotropic medium along the x axis the diffusion equation has the form

$$\frac{\partial C}{\partial \tau} = D_{\text{eff}} \frac{\partial^2 C}{\partial x^2}. \quad (1)$$

It should be noted that this equation describes not only processes in which the coefficient of effective diffusion D_{eff} is constant but also processes where it depends on the time τ .

To solve the problem of drying hygroscopic materials, usually a system of equations of hydrodynamics and heat and mass transfer is set up and solved numerically [1-3]. To describe the process of the nonstationary diffusional kinetics of chemical reactions an approximate quasistationary method, or the method of an equally accessible surface, is suggested in [2]. It is applicable to cases where all portions of the surface are equally accessible for diffusion, in particular, to an isotropic medium. The quasistationary method combines the results of integration of diffusion equation (1) with a simple boundary condition ($C = 0$ on the surface) and the results of processing experimental data by the Newton equation for mass transfer. Thus, in accordance with the quasistationary method, the diffusion flux J must correspond to the mass-transfer equation and must be proportional to the current concentration C on the surface of the material:

$$J = \beta (C_0 - C) = kC, \quad (2)$$

where C_0 is the initial concentration in the material. From Eq. (2) the dimensionless concentration, which describes the kinetics of the process, is obtained:

$$\frac{C}{C_0} = \frac{1}{1 + \frac{k}{\beta}}. \quad (3)$$

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Computational Equations. It should be noted that the above dependences (2) and (3) were obtained for nonstationary chemical kinetics, and they cannot be used directly for describing diffusional processes in a heterogeneous system, since for the time $\tau \rightarrow \infty$ they give a zero value for the dimensionless concentration. At the same time, in the processes considered by us the concentration in the material tends to the equilibrium value C^* as $\tau \rightarrow \infty$, and the dimensionless concentration tends to the finite value C^*/C_0 .

In our case the initial condition is the distribution of the concentration at the initial instant of time:

$$C = C_0 \quad \text{when} \quad \tau = 0, \quad (4)$$

and the boundary condition is the proportionality of the diffusion flux to the concentration on the surface of the material:

$$J = D_{\text{eff}} \left. \frac{\partial C}{\partial \tau} \right|_{x=0} = kC. \quad (5)$$

To use relations (2) and (3), it is appropriate to replace the parameter C/C_0 , as in [4], by the dimensionless combination $(C - C^*)/(C_0 - C^*)$, which complies with the above-indicated limiting values. Thus, at $\tau = 0$ we have $C = C_0$, $C/C_0 = 1$, and the process proceeds in the kinetic region ($k = 0$). As $\tau \rightarrow \infty$, the concentration in the material tends to the equilibrium value C^* , and the dimensionless combination tends to zero, since the process proceeds in the diffusional region ($\beta = 0$).

It can easily be seen that the dimensionless concentration C/C_0 can be found in terms of the dimensionless combination from the following relation:

$$\frac{C}{C_0} = \frac{C - C^*}{C_0 - C^*} \frac{C_0 - C^*}{C_0} + \frac{C^*}{C_0}. \quad (6)$$

This combination can also be obtained directly in deriving equations for describing the kinetics of diffusion processes in a heterogeneous system by means of a modified quasistationary method.

According to the mass-transfer equation, for the quasistationary kinetics in a heterogeneous system the diffusional flux is proportional to the difference between the initial C_0 and current C concentrations, and at the same time it is proportional to the surface concentration. For desorption, the latter must be smaller than in the bulk of the material C . We assume it to be equal to the difference between the current and equilibrium concentrations $C - C^*$, and then similarly to Eq. (2), for the flux we have

$$J = \beta (C_0 - C) = k (C - C^*). \quad (7)$$

After the simplest transformations, Eq. (7) yields an expression similar to Eq. (3):

$$\frac{C - C^*}{C_0 - C^*} = \frac{1}{1 + \frac{k}{\beta}}. \quad (8)$$

In [2] it is shown that for nonstationary diffusion in a stationary medium the mass-transfer coefficient depends on time in the following way:

$$\beta = \sqrt{\left(\frac{D_{\text{eff}}}{\pi \tau} \right)}. \quad (9)$$

Substituting the expression for β into Eq. (8), we obtain

$$\frac{C - C^*}{C_0 - C^*} = \frac{1}{1 + k \sqrt{\left(\frac{\pi \tau}{D_{\text{eff}}} \right)}}. \quad (10)$$

Introducing into Eq. (10) the quantity $\sigma = D_{\text{eff}}/\pi k^2$, which is the characteristic time, and the parameter of the dimensionless time of the process τ/σ , similarly to what was adopted by us earlier [5], we obtain a description of the kinetics in the form

$$\frac{C - C^*}{C_0 - C^*} = \frac{1}{1 + k \sqrt{\left(\frac{\tau}{\sigma}\right)}}, \quad (11)$$

whence for the time of the process we have

$$\tau = \sigma \left[\frac{C_0 - C^*}{C - C_0} - 1 \right]^2. \quad (12)$$

It should be noted that Eqs. (11) and (12) are applied to the process of desorption in a stationary medium. With desorption in the liquid phase, one succeeds in carrying out the process as in a stationary medium, whereas in the gas phase, which has a considerably lower viscosity, this is usually impossible. It is suggested that the kinetics of the process in a moving medium be described by the following power-law dependence:

$$\frac{C - C^*}{C_0 - C^*} = \frac{1}{1 + \left(\frac{\tau}{\sigma}\right)^n}. \quad (13)$$

A check of relation (13) in carrying out a comparison with a numerical solution of diffusion equation (1) showed that deviations lie within the accuracy of experiment (up to 5%). With allowance for Eq. (13), the corresponding expression for the time of the process acquires the form

$$\tau = \sigma \left[\frac{C_0 - C^*}{C - C_0} - 1 \right]^{1/n}. \quad (14)$$

Differentiating Eq. (14) with respect to time, we obtain an expression for the rate of drying N :

$$-\frac{dC}{d\tau} = \frac{n(C_0 - C^*) \left(\frac{\tau}{\sigma}\right)^{n-1}}{\sigma \left[1 + \left(\frac{\tau}{\sigma}\right)^n\right]^2}. \quad (15)$$

To find the change in the temperature in the process of drying, we use a relation that was suggested for a plane material in [6] with account for the expression for the gradient of the rate of drying

$$\psi = \frac{N}{N_0}, \quad (16)$$

where N_0 is the initial rate of drying. According to [6], the temperature relation has the form

$$t = t_w + (t_d - t_w - t_1)(1 - \psi), \quad (17)$$

where t_d and t_w are the temperature of a dry and a wet thermometer, respectively.

To obtain the temperature relation for drying a bulk material, we introduce a coefficient a that takes account of the temperature gradient within the material:

$$t = t_w + (t_d - t_w - t_1)(a - \psi). \quad (18)$$

Comparison with Experiment. Normally the description of the kinetics of drying of nonhygroscopic materials, for example of mineral origin, is not difficult. In this respect the most interesting is the drying of

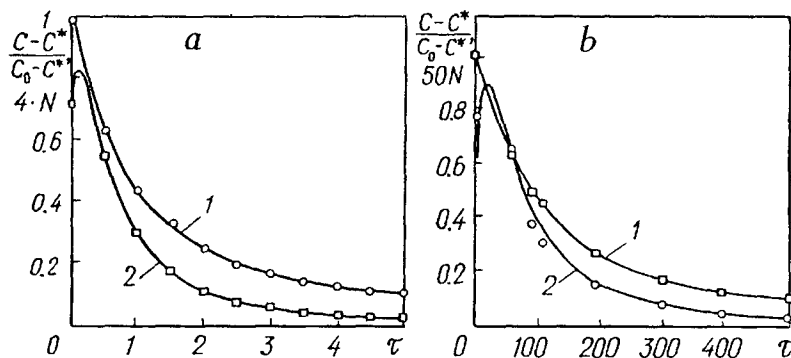


Fig. 1. Kinetics of the change in the dimensionless combination of the moisture content $(C - C^*) / (C_0 - C^*)$ (1) and rate of the process (1/h (a) and 1/min (b)) (2) with time τ in drying corn grains (a) and wool fiber (b).

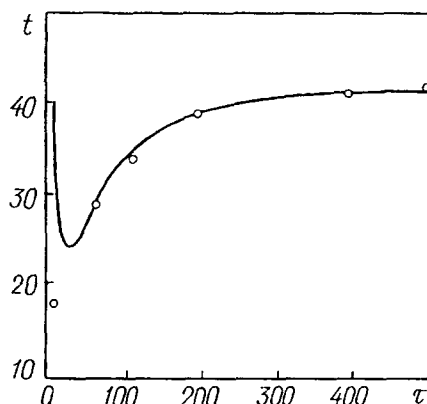


Fig. 2. Dependence of the change in the temperature t ($^{\circ}\text{C}$) in the central zone of a bobbin of wool fiber on the time of the process τ (min).

hygroscopic fibrous and granular materials of organic and plant origin that have a developed structure of pores and capillaries. We consider drying of such materials on the example of wool fiber and grains of corn.

Figure 1a presents experimental data [3] on drying a thin layer of corn grains with an initial moisture content of 0.31 kg/kg of dry material by hot air at a temperature of 71°C and $\varphi = 0.118$ and a comparison of these data with a calculation based on the kinetics of drying (Eq. (13), curve 1) and the drying rate (Eq. (15), curve 2); the values are multiplied by 4 to use the scale of concentrations. The initial rate of drying is 0.18 1/h. The curves are constructed for the values $\sigma = 0.785$ and $n = 1.21$. On the curves one can see a short period of constant rate of drying up to $\tau \sim 0.8$ h and a long subsequent period of drying. The experimental data correspond satisfactorily to the computational curves, except for the initial segment on the curve for the rate of the process. Although the curve for the change in the moisture content agrees well with the experiment, at $\tau \approx 20$ min an inflection point appears that is barely noticeable on the graph. Here the curve of the rate of drying has a sharp bend (an inflection point). It is worth noting that there is a similarity between the curves of the change in the moisture content and the rate of drying, which is typical for solutions of the diffusion equation with a boundary condition in the form of a diffusional flux.

Figure 1b presents experimental data [3] on drying wool fiber with an initial moisture content of 2.64 kg/kg on a Bakelite bobbin of height 15 cm and outer diameter 7.36 cm by hot air at a temperature of 78.5°C and $\varphi = 0.03$ and a comparison of these data with the predicted kinetics of drying (Eq. (13), curve 1) and rate of drying (Eq. (15), curve 2); the values are multiplied by 50 to use the scale of concentrations. The initial rate of drying is 0.0153 1/min. Both curves are constructed for the values $\sigma = 85$ and $n = 1.3$. On the curves one also sees a short period where the rate of drying is constant (up to the value $\tau \sim 70$ min) and a subsequent long period of drying. The experimental data correspond satisfactorily to the computational curves, except for the initial portion of the curve for the rate, just as in the case of drying corn. We also see a similarity between the curves of the change in

the moisture content and the rate of drying. Therefore the equation for the rate of drying can be represented in the form of dependence (13) with the values $\sigma = 2.08$ and 1.32 (not shown in Fig. 1b). Even though there is no sharp bend in the rate here, the curve does not coincide with the value of the rate N_0 .

A check of proposed relation (13) in a comparison with a numerical solution for the kinetics of drying corn and wool given in [3] showed that the curves of drying virtually coincided with the numerical calculation.

The experimental dependence of the temperature for the central zone of a bobbin of wool fiber on the time of drying [3] is shown in Fig. 2. The temperature changed from $18\text{ }^{\circ}\text{C}$ at the beginning to $48.5\text{ }^{\circ}\text{C}$ at the end of the process. For air the temperature of a dry thermometer was $t_d = 78.5\text{ }^{\circ}\text{C}$ and of a wet one $t_w = 32\text{ }^{\circ}\text{C}$, whence $t_1 = 30\text{ }^{\circ}\text{C}$. In processing the experimental data presented relation (18) is obtained with the coefficient $a = 0.618$. Comparison of it with the experimental data shows rather good agreement, except for the initial portion. An explanation of this fact is given by analysis of the kinetics of drying.

CONCLUSIONS

1. The article considers the description of the kinetics of drying of hygroscopic materials by a modified quasistationary method obtained in analyzing the diffusional flux in the material. The final equations are rather simple and agree with results of a numerical solution of the system of hydrodynamic and heat and mass transfer equations.

2. Using, as an example, the drying of wool fiber and corn grains, the application of the proposed equations to calculation of the kinetics of the process is shown. Satisfactory agreement is obtained between the prediction of the loss of moisture, the rate of the process, and the change in the temperature with the time of the process and experiment, except for the initial portion of the constant rate of drying.

3. The proposed equations may be used to calculate the process of drying of hygroscopic materials.

NOTATION

C , moisture content per kg of dry material; C^* , equilibrium moisture content; D_{eff} , coefficient of effective diffusion; J , diffusion flux; k , kinetic coefficient; n , coefficient in Eq. (13); t , temperature; x , current coordinate; φ , relative humidity of the air; β , coefficient of mass transfer; σ , characteristic time; τ , current time of the process; t_1 , difference of the temperatures of the heat carrier and the material at the end of drying; ψ , gradient of the rate of drying. Subscripts: 0, initial; d, dry; w, wet.

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